Subject: Leaving Certificate Maths

Teacher: Mr Murphy

Lesson 7: Geometry I

7.1 Learning Intentions

After this week's lesson you will be able to;

- Use the correct language when discussing geometry
- Apply content from the theorems of geometry to solve problems

7.2 Specification

Students learn about	Students working at OL should be able to	In addition, students working at HL should be able to
2.1 Synthetic geometry	– perform constructions **16-21** (see *Geometry for Post-primary School Mathematics*) – use the following terms related to logic and deductive reasoning: theorem, proof, axiom, corollary, converse, implies – investigate theorems 7, 8, 11, 12, 13, 16, 17, 18, 20, 21 and corollary 6 (see Geometry for Post-primary School Mathematics) and use them to solve problems	– perform construction **22** (see *Geometry for Post-primary School Mathematics*) – use the following terms related to logic and deductive reasoning: is equivalent to, if and only if, proof by contradiction – prove theorems 11,12,13, concerning ratios (see Geometry for Post-primary School Mathematics), which lay the proper foundation for the proof of the theorem of Pythagoras studied at junior cycle

7.2 Specification

Student should practise different ways of solving problems, building up their arsenal of technique on familiar problems will help them to tackle unfamiliar ones. Students at Higher level and Ordinary level should pay particular attention to algebraic methods of solving problems, as such method are directly examinable at these levels.

7.4 Foundations of Geometry

A Plane: A 2-D space that has a width and length but no depth.

A Point: A precise location in a plane with no magnitude.

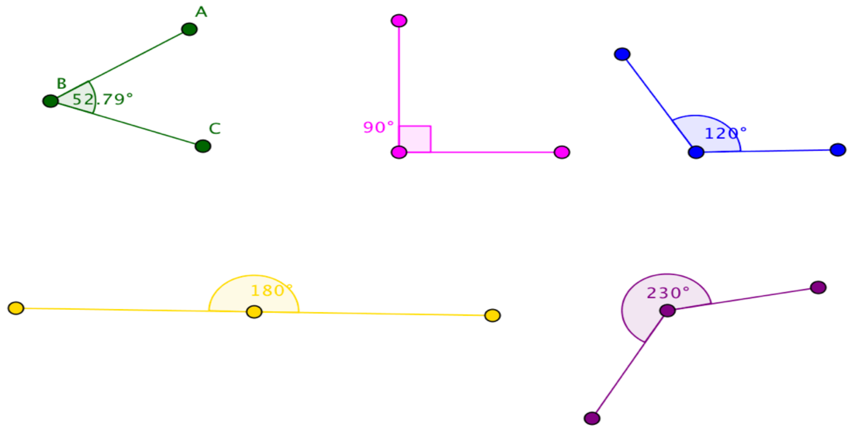
Coplanar Points: Two points in the same plane.

A Line: A collection of points extending to infinity in both directions AB.

A Line Segment: A portion of a line with a start and an end point [AB].

A Ray: A line with a start point but no end point.

Angles:



Axioms:

An axiom is a statement that we accept as true without any formal proof

Axiom 1: There is only one straight line that joins two points

Axiom 2: This is the properties of the distance of a line

- Distance is never negative
- $|AB| = |BA|$
- $|AB| = |AC| + |CB|$

Axiom 3: Properties of the measure of an angle

- Straight angle =180°
- Degrees around a point =360°

Theorems:

A **Theorem** is a rule that has been proved by following a list of steps known as a proof

A **Corollary** is a statement that follows directly from a previous proof for example:

Corollary No. 3: Each angle in a semi-circle is 90°

The **Converse** of a Theorem is the reverse of the theorem

When dealing with the converse it is best to split the theorem up into two pats:

Statement: If A, then B

Converse: If B, then A

Example: **S:** If a man was born in Dublin, then he is Irish.

C: If a man is Irish, then he was born in Dublin.

Proofs:

A Mathematical proof is a number of steps that show a theorem to be true. They usually have the following headings

Theorem:

Given:

To prove:

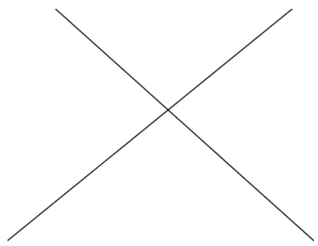
Construction:

Proof:

7.5 Theorems

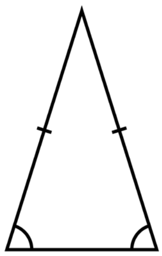
Theorem 1:

Vertically opposite angles are equal in measure



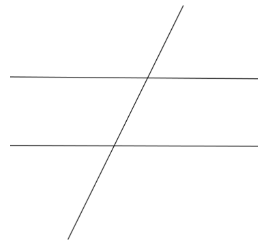
Theorem 2:

In an isosceles triangle the angles opposite the equal sides are equal in measure.



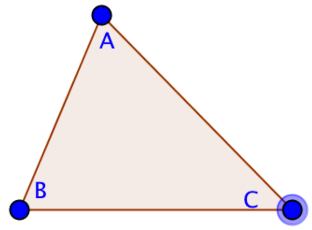
Theorem 3:

If a transversal makes equal alternate angles on two lines, then the lines are parallel.



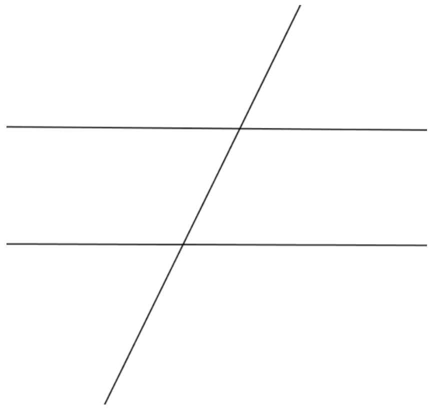
Theorem 4:

The angles in any triangle are sum to 180°



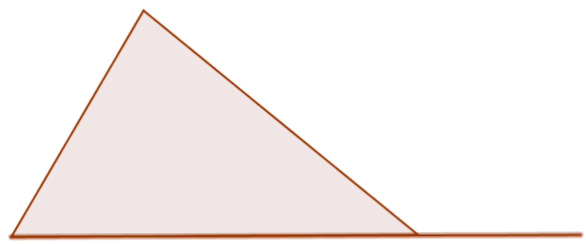
Theorem 5:

Two lines are parallel if for any transversal, the corresponding angles are equal.



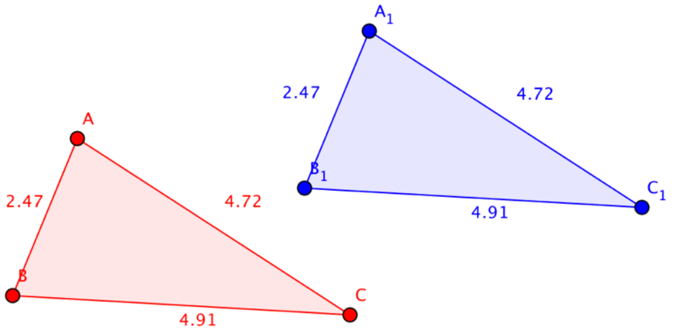
Theorem 6:

Any exterior angle of a triangle is equal to the sum of the two opposite interior angles



7.6 Congruent Triangles

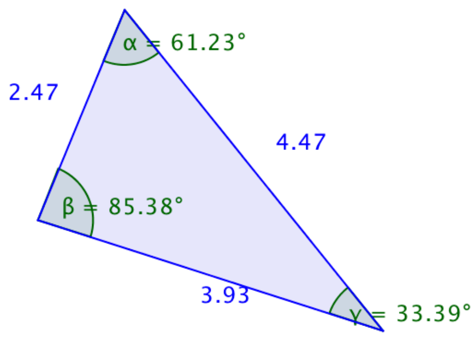
Congruent (≡) means identical



From the video, write down the conditions for triangles to be congruent below

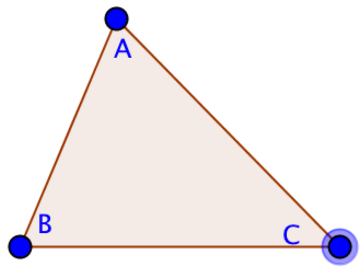
<u>Theorem 7:</u>

The largest angle is opposite the largest side

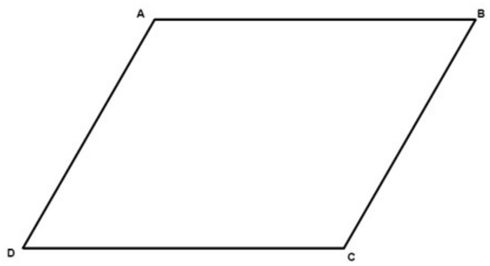


Theorem 8:

The sum of two sides of any triangle are always greater than the 3rd side.

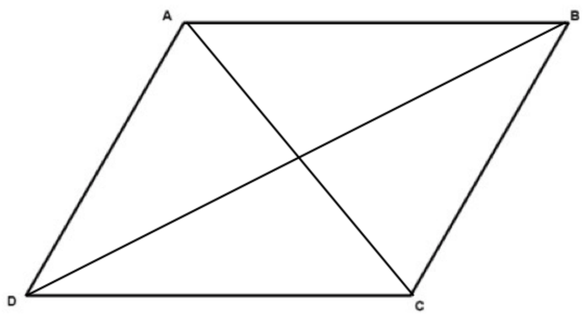


Theorem 9:



Opposite angles are equal
Opposite sides are equal
All angles sum to 360°

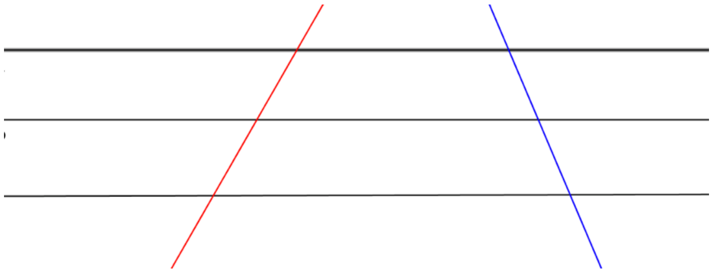
Theorem 10:



The diagonals of a parallelogram bisect each other.

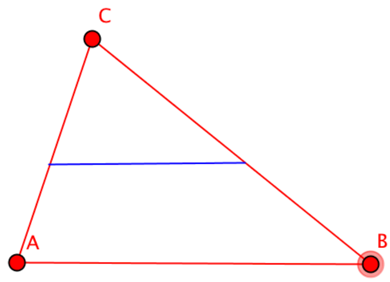
Theorem 11:

If 3 parallel lines cut a transversal into two equal parts, then the same is true for any other transversal.



Theorem 12:

In a triangle ABC, if there is a parallel line to [AB], that cuts [AC] into the ratio d:e, then it also cuts [BC] into the same ratio.



$$\frac{|AX|}{|XC|} = \frac{|BY|}{|YC|}$$

$$\frac{Top\ left\ length}{|Bottom\ Left\ length|} = \frac{Top\ right\ length}{Bottom\ right\ length}$$

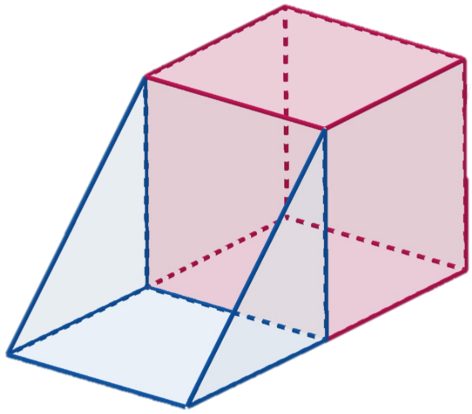
7.7 Recap of Learning Intentions

After this week's lesson you will be able to;

- Use the correct language when discussing geometry
- Apply content from the theorems of geometry to solve problems

7.8 Homework Task

Using what you have learned this week, label the below diagram correctly and prove that the two blue triangles are congruent. The blue surface is a drop-down replica of the front face of the cube



7.9 Solutions to 6.12

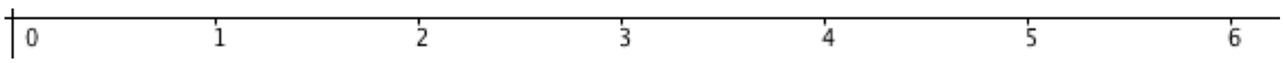
1) $5(2x-1) > 2(x-1)+5, x \in \mathbb{N}$....don't forget to graph solutions on number line.

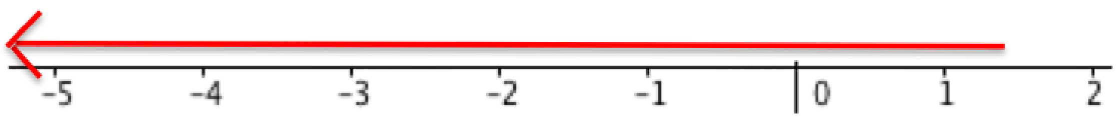
$10x - 5 > 2x - 2 + 5$

$10x - 5 > 2x + 3$

$8x > 8$

$x > 1$





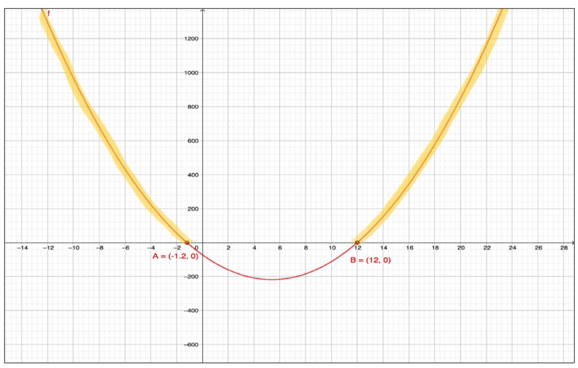
2) $-72 \geq -5x^2 + 54x$

$0 \geq -5x^2 + 54x + 72$

$0 \leq 5x^2 - 54x - 72$

$0 \leq (x-12)(5x+6)$

$x = 12 \; and \; x = -\frac{6}{5}$



$-\frac{6}{5} > x > 12$

3) $\frac{3x-7}{x-4} < 2, x \neq 4$

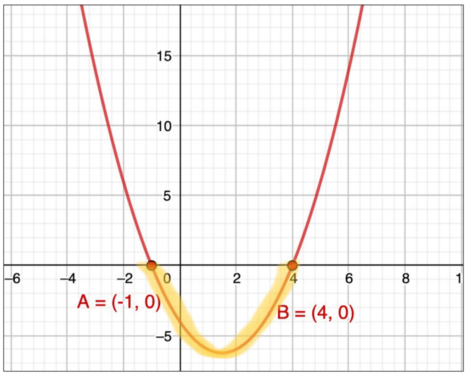
$\frac{(x-4)^2 3x-7}{x-4} < 2(x-4)^2$

$(x-4)(3x-7) < 2\left(x^2 - 8x + 16\right)$

$3x^2 - 12x - 7x + 28 < 2x^2 - 16x + 32$

$x^2 - 3x - 4 < 0$

$(x-4)(x+1) < 0$



$-1 < x < 4$

4) Prove that for all values of $\mathbb{R}$ $(a-2)x^2 + 2x - a = 0$ has real roots

$b^2 - 4ac \geq 0$

$(2)^2 - 4(a-2)(-a) \geq 0$

$4 - 4(-a^2 + 2a) \geq 0$

$4 + 4a^2 - 8a \geq 0$

$4a^2 - 8a + 4 \geq 0$

$(2a-2)(2a-2) \geq 0$

$(2a-2)^2 \geq 0$ always true as any real number squared is a positive number i.e. ≥ 0.